Small-signal Analysis of Darlington Pair

To determine the effective small I/P resistance from B to E function.

$$r_{e_2} = \frac{26mV}{I_{E_2}}$$

looking into the base of transistor Q_2

$$r_{in(Base)_2} = \beta_2 r_{e_2}$$

$$r_{e_1} = \frac{26mV}{I_{e_1}} \qquad I_{E_2} = (\beta_2 + 1) I_{B_2}$$

$$\text{or } I_{E_2} = (\beta_2 + 1) I_{E_1}$$

$$I_{E_1} = \frac{I_{E_2}}{\beta_2} \qquad \text{or } \frac{I_{E_2}}{I_{E_1}} = \beta_2$$

$$r_{e_1} = \beta_2 \left(\frac{26mV}{I_{E_2}} \right)$$

Now,

~~$r_{in\,Dp} = \beta_1 r_{e_1} + \beta_2 r_{e_2}$~~

~~$= \beta_1 * \beta_2 \left(\frac{26mV}{I_{E_2}} \right)$~~

$$r_{in_{DP}} = \beta_1 (r_{e_1} + r_{in\,Base_2})$$

: ~~$\beta_1 (r$~~

It is the effective resistance looking into the base of Q_1 across the effective small signal input resistance

composite base to emitter terminal. $r_{in\,DP}$ is the effective small signal effect of Darlington pair

$$r_{in\,Dp} = \beta_1 (r_{e_1} + \beta_2 r_{e_2})$$

$$= \beta_1 (\beta_2 r_{e_2} + \beta_2 r_{e_2})$$

$$= 2\beta_1 \beta_2 r_e$$

$$r_{e(Dp)} = \frac{r_{in\,Dp}}{\beta_{Dp}}$$

When a load is connected at the output of the Darlington pair configuration then the total input resistance to the follower circuit is,

$$r_{in} = r_{in\,DP} + \beta_{DP} R_L$$

Memod of coupling / Types of Multistage Amplifier

1. R-C coupled Amplifier
2. Transformer coupled Amplifier
3. Direct coupled Amplifier

Chapter: 2

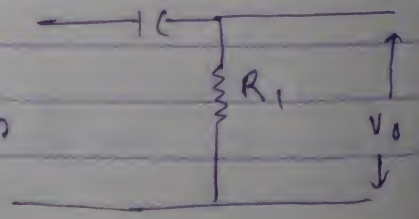
Freqvency Response of the coupled Amplifier.

→ Curve drawn between voltage gain & Signal freqvency response - Performance Analysis.

→ Inorder to Know the uniformity of the designed amplifier.

→ Freq characteristic divides: Midband & upper frequency

low freqvency Response:-



In the low freq region ie below mid band.

$T_1 = R_1 C_1$

current thrvgh ckt

$I_1 = \frac{vin}{R - jX_1}$, capacitive reactance,

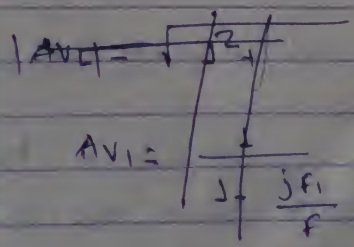
o/p voltage gain, $V_0 = I_1 R_1$

$= \frac{Vin}{R - jX_1} * R_1$

$\therefore V_0 / V_{in} = \frac{R_1}{R - jX_1}$

$A_{V_1} = \frac{V_0}{V_{in}} = \frac{1}{1 - \frac{j}{2\pi f_c R_1}}$

$= \frac{1}{1 - \frac{jf_1}{f}}$; where $f_1 = \frac{1}{2\pi C_1 R_1}$: cut off freq.



$|A_{VL}| = \frac{1}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}}$

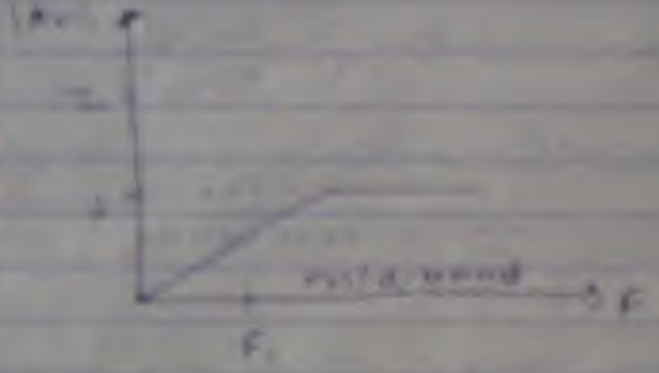
$\theta = \tan^{-1}\left(\frac{f_1}{f}\right)$

Conditions:

at $f = f_1$ $|A_{VL}| = \frac{1}{\sqrt{2}} \times 0.707$

at $f = 0$ $|A_{VL}| = 0$

at $f = \infty$ $|A_{VL}| = 1$



Lower cut off frequency (f_1)

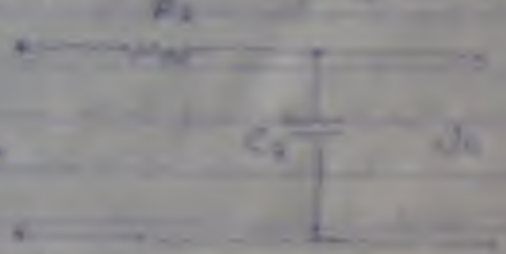
Mid band region starts at $f > f_1$, $A_{VL} \to 1$

f_1 is the frequency at which the gain has reduced to 0.707 times the midband

Value (A_{VMB}) which is equal to $\frac{1}{\sqrt{2}}$ ($\frac{1}{\sqrt{2}}$) or -3 dB (reduction). So f_1 is referred to as lower 3dB freq or cut off frequency

Note: At low frequency C.C offers considerable resistance which causes the voltage drop due to this drop the input of the next stage reduces and hence the overall voltage gain reduces

High Frequency Response



$$V_{out} = \frac{1}{j2\pi f C_2}$$

$$R_2 = \frac{1}{j2\pi f C_2}$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + j2\pi f C_2 R_2}$$

$$A_V = \frac{1}{1 + j2\pi f C_2 R_2}$$

$$A_V = \frac{1}{1 + j\frac{f}{f_2}} \quad \text{where } f_2 = \frac{1}{2\pi C_2 R_2}$$

upper cut-off frequency

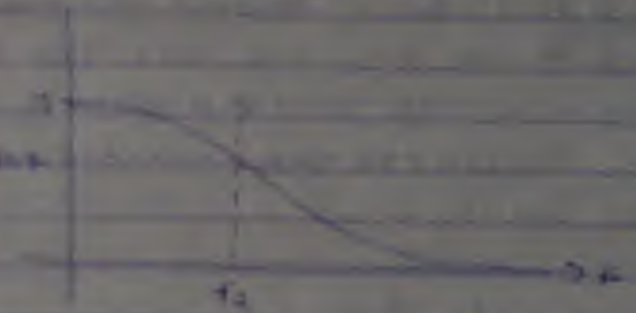
$$|A_V| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_2}\right)^2}}$$

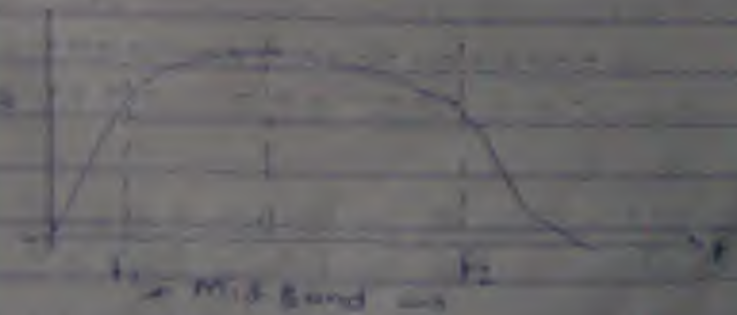
$$\theta = \tan^{-1}\left(\frac{f}{f_2}\right)$$

at $f = 0$, phase = 0

at $f = f_2$, phase = $\pi/4$

at $f = \infty$, phase = $\pi/2$





Imp

Why the gain reduces at high frequency?

They stray capacitance Cs causes the output voltage of first stage to be shorted ~~to~~ to ground and the physical capacitance of active device present in the amplifier cannot act as open ckt. They posses some reactance. Due to this, part of output voltages feed back to the input via Cbc and the output voltage will be grounded via CCE. Hence the gain reduces.

Bandwidth (B/W):

The differences between two extreme frequencies upper cut off (f2) and lower cut off (f1) or it is the band of frequencies where the mid band gain Avm=1

or, It is define as the range of frequencies for which the gain falls 70.7% of its maximum valve ie the attenuation is -3db.

i.e; gain at cut off 70.7% of Avm

(Q) Why cut-off frequencies are called 3dB or half-power frequencies?

let Vm be the midband valve.

$\Rightarrow P_{mid\text{-}band} = \frac{V_m^2}{R}$

at cut off $= \frac{V_m}{\sqrt{2}}$

and $P_{cut\ off} = \left(\frac{V_m}{\sqrt{2}}\right)^2 \cdot \frac{1}{R} = \frac{V_m^2}{2R}$

$= \frac{P_{mid\ band}}{2}$

Hence cut-off frequencies called half-power frequencies.

So power in dB;

~~20 log (power~~

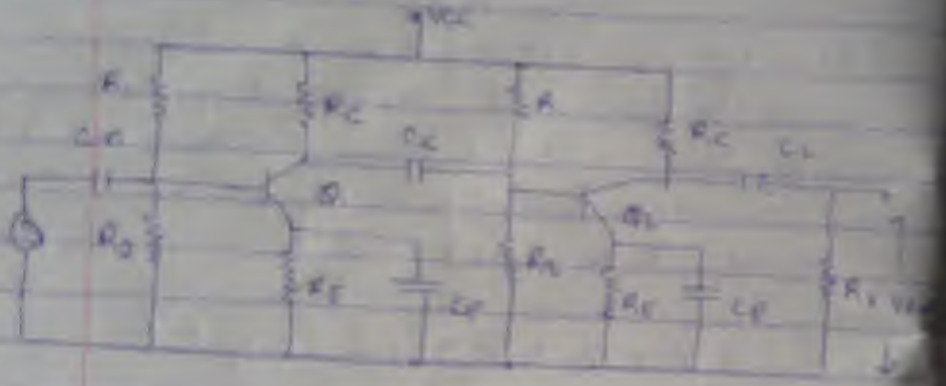
$20\log(P_{cut\ off}) = 10\log\left(\frac{P_{mid\text{-}band}}{2}\right)$

$= 10\log(P_{mid\text{-}band}) + 10\log\frac{1}{2}$

$= 10\log(P_{mid\text{-}band}) - 3dB$

Hence the gain at cut off frequencies are 3dB less then the gain at mid-band range.

R-C coupled Amplifier -



C_C – couples the ac signal voltage to the base of Q_1

else,

$V_{CC} \parallel V_{BB}$

so can allow ac-signal to flow in the ckt.

→ Two transistors are identical and use a common source of voltage (power supply) V_{CC}.

→ R_1, R_2 and R_E form biasing and stabilization ckt.

→ Here in this arrangement the signal developed at collector (R_L) of the 1st

stage is coupled to the base of 2nd stage through C_C & blocking of dc component.

Input resistance of 2nd transistor is in parallel with R_2 and hence effective voltage across R_2 is reduced hence reducing the gain.

Analysis: For mid-frequency range

→ X_{C_C} very small i.e the load impedance

→ Since it very large it considered as open ckt.

→ R_1 and R_2 are very large in comparison to R_E

→ C_E very large i.e X_{C_E} negligible as short ckt.

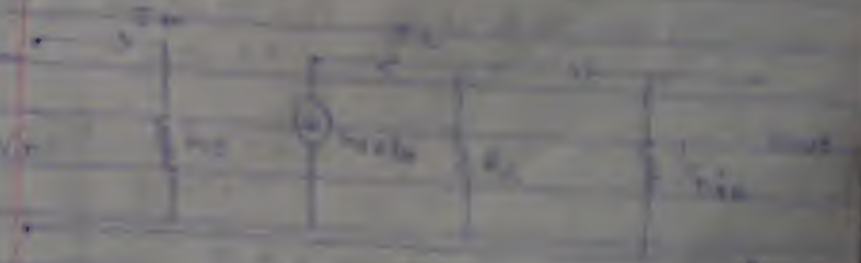


Fig Approximate equivalent ckt of R-C coupled transistor amplifier

Mid Band Frequency Analysis

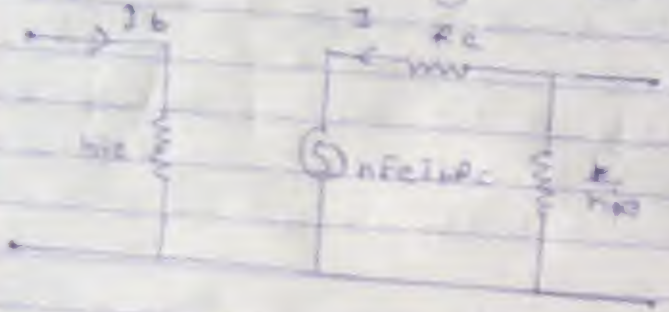


fig: Thevenin equivalent ckt.

from above:-

$$I = \frac{h_{fe} I_b R_c}{R_c + h_{ie}} \quad -(i)$$

Current gain (Aim) $= \frac{I}{I_b} = \frac{h_{fe} I_b R_c}{R_c + h_{ie}} * \frac{1}{I_b}$

$$= \frac{h_{fe} R_c}{R_c + h_{ie}} \quad -(ii)$$

output voltage, Vout $= h_{ie} * I$

$$= h_{ie} * \frac{h_{fe} I_b R_c}{R_c + h_{ie}} \quad -(iii)$$

Input voltage, $V_{in} = h_{ie} * I_b$

Voltage gain, (Avm) $= \frac{V_{out}}{V_{in}}$

$$= \frac{h_{ie} h_{fe} I_b R_c}{R_c + h_{ie}} * \frac{1}{h_{ie} I_b}$$

$$= \frac{h_{fe} R_c}{R_c + h_{ie}} \quad -(iv)$$

From equation (ii) and (iv) Aim = Avm
In this case, the impedance offered by Cc is comparable to the Rc so necessary to include in the equivalent ckt

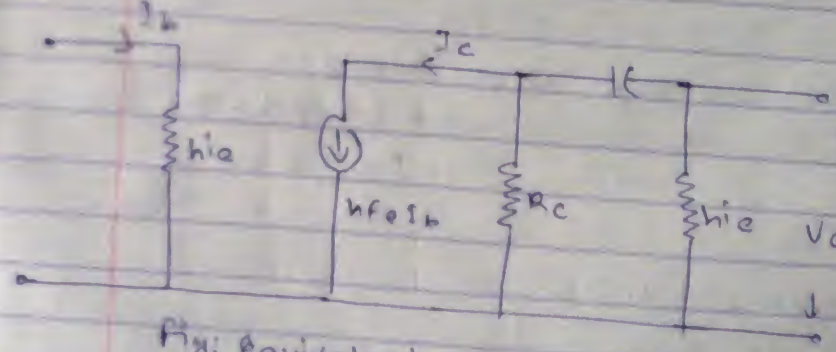


fig: Equivalent ckt for low frequency range.

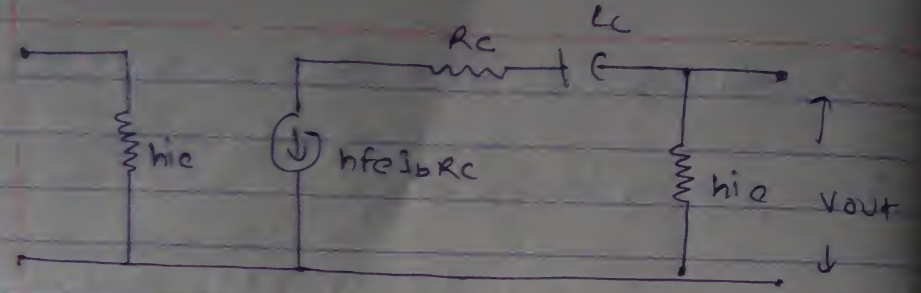


Fig: Thevenin Equivalent

Now,

$$I = \frac{h_{fe} I_b R_c}{R_c + h_{ie} + \frac{1}{j\omega C}}$$

current gain, $A_{ib} = \frac{I}{I_b} = \frac{h_{fe} R_c}{R_c + h_{ie} + \frac{1}{j\omega C_c}}$ — (i)

Output voltage, $V_{out} = h_{ie} \cdot I$

$$= \frac{h_{ie} h_{fe} R_c I_b}{R_c + h_{ie} + \frac{1}{j\omega C_c}}$$

$V_{in} = h_{ie} \cdot I_b$

Voltage gain, $A_{VL} = \frac{V_{out}}{V_{in}} = \frac{h_{fe} R_c}{R_c + h_{ie} + \frac{1}{j\omega C_c}}$

$\therefore A_{iL} = A_{VL}$

High freq. Analysis:

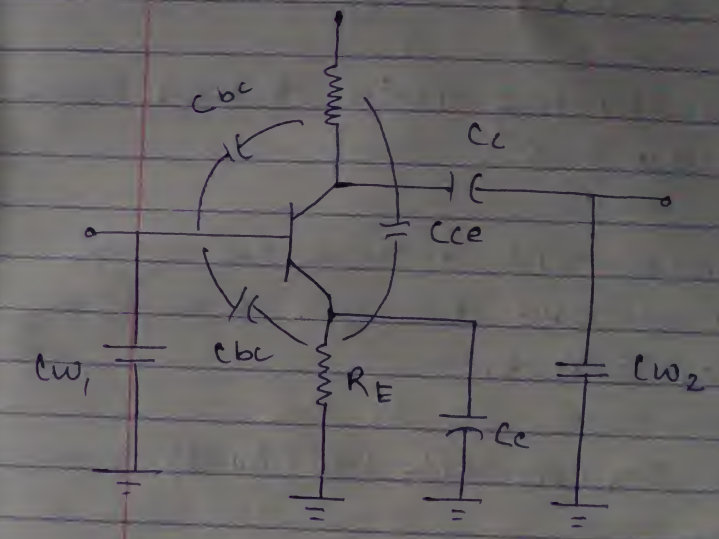


Fig: R-C coupled Amp at high freq.

C_{w_2} = wiring capacitance between connecting wire of ckt

C_{bc}:- causes (-ve) feedback in the ckt so gain is reduce. feedback effect (↑) with the (↑) in freqvency Because (↑) in freq (↓) reactive impedance of capacitor.

Cbe: Offers low impedance path at i/p side at high frequency Reducing the i/p impedance and i/p signal.

Cce - Produces shunting effect at high freq. of on the o/p sides.

Cbc: feedback effect occured from o/p ckt to i/p ckt through Cbc is called miller effect.

Statement: $C_{in(miller)} = C_{bc}(A_v+1)$

Here, Cbe and Cbc may be replaced with a capacitor 'Cd' across the input resistance hie of the transistor ie the capacitance $\frac{1}{\omega C_d}$ will have appreciable shunting effect on Rc and hie.

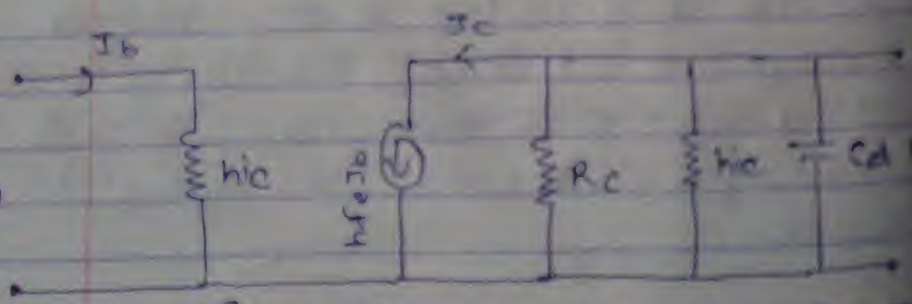


fig: Equivalent ckt for high freq.

fig: Equivalent ckt

Now,

current, $I = \frac{h_{fe} I_b (R_c // h_{ie})}{(R_c // h_{ie}) + \frac{1}{j\omega C_d}}$

current gain, $A_{is} = \frac{I}{I_b}$

$= \frac{h_{fe}(R_c // h_{ie})}{(R_c // h_{ie}) + \frac{1}{j\omega C_d}}$

output voltage, $V_{out} = \frac{1}{j\omega C_d} * I$

$= \frac{1}{j\omega C_d} * \frac{h_{fe} I_b (R_c // h_{ie})}{(R_c // h_{ie}) + \frac{1}{j\omega C_d}}$

$V_{in} = I_b * h_{ie}$

Now,

$$A_{VM} = \frac{V_o}{V_{in}} = \frac{h_{fe} I_b (R_c // h_{ie})}{j\omega C_d \left[R_c // h_{ie} + \frac{1}{j\omega C_d}\right]} * \frac{1}{I_b \cdot h_{ie}}$$

$$= \frac{h_{fe}(R_c // h_{ie})}{j\omega C_d \cdot h_{ie}\left[(R_c // h_{ie}) + \frac{1}{j\omega C_d}\right]}$$

$$= \frac{h_{fe} R_c}{h_{ie} R_c j\omega C_d + R_c + h_{ie}}$$

So with the increase in input frequency the magnitude of the voltage gain falls.

* lower cut-off frequency:-

The cut off frequency 'f_1' may be defined as the frequency at which the $|A_{VL}|$ falls to $1/\sqrt{2}$ or 0.707 times the maximum gain at mid band range (A_{VM})

Then,

at $f = f_1$

$$|A_{VL}| = 1/\sqrt{2}\ |A_{VM}|$$

or, $$\left|\frac{A_{VL}}{A_{VM}}\right| = 1/\sqrt{2}$$

We have,

$$|A_{VL}| = \frac{h_{fe} R_c}{\sqrt{(h_{ie} + R_c)^2 + \left(\frac{1}{2\pi f C_c}\right)^2}}$$

And,

$$|A_{VM}| = \frac{h_{fe} \cdot R_c}{h_{ie} + R_c}$$

Then,

$$\frac{|A_{VL}|}{|A_{VM}|} = \frac{1}{\sqrt{1 + \left\{\frac{1}{2\pi f C_c (h_{ie} + R_c)}\right\}^2}} \quad \text{--- (A)}$$

If f_1 is the lower cut off freq. then,

$$1/\sqrt{2} = \frac{1}{\sqrt{1 + \left\{\frac{1}{2\pi f_1 C_c}(h_{ie} + R_c)\right\}^2}}$$

Sq. on both sides,

$$1/2 = \frac{1}{1 + \left\{\frac{1}{2\pi f_1 C_c}(h_{ie} + R_c)\right\}^2}$$

$$\text{or, } 1 + \left(\frac{1}{2\pi f_1 C_c (h_{ie} + R_c)}\right)^2 = 2$$

$$\text{or, } \left(\frac{1}{2\pi f_1 C_c (h_{ie} + R_c)}\right)^2 = 1$$

$$\text{or, } 1 = 2\pi f_1 C_c (h_{ie} + R_c)$$

$$\therefore f_1 = \frac{1}{2\pi C_c (h_{ie} + R_c)} \quad \text{— (B)}$$

From A & B

$$|A_{VL}| = \frac{|A_{VM}|}{\sqrt{1 + \left(\frac{f_1}{f}\right)^2}} = \frac{|A_{VM}|}{\sqrt{1 + \tan^2\theta_L}}$$

where $\tan\theta_L = \frac{f_1}{f}$

$$\theta_L = \tan^{-1}\left(\frac{f_1}{f}\right)$$

i.e; IF θ_L T F↓

θ_L → phase Response of the transistor.

Higher-cut off frequency:
The cut-off frequency 'f_2' may be defined as the frequency at which the $|A_{VH}|$ falls to $1/\sqrt{2}$ or 0.707 times the maximum gain at mid band range (A_{VM})

Thus at $f = f_2$

$$|A_{VH}| = 1/\sqrt{2}\,|A_{VM}|$$

$$\text{or, } \left|\frac{A_{VH}}{A_{VM}}\right| = 1/\sqrt{2}$$

Now,

$$|A_{VH}| = \frac{h_{fe} R_c}{(h_{ic} + R_c)\sqrt{1 + \left(\frac{2\pi f C_d h_{ic} R_c}{h_{ie} + R_c}\right)^2}}$$

&

$$A_{VM} = \frac{h_{Fe} R_c}{h_{ic} + R_c}$$

Now,

$$\left|\frac{A_{VH}}{A_{VM}}\right| = \frac{1}{\sqrt{1 + \left(\frac{2\pi f C_d h_{ic} R_c}{h_{ie} + R_c}\right)^2}} \quad \text{— (C)}$$